Davies theory for reservoir-induced entanglement in a bipartite system

2007 J. Phys. A: Math. Theor. 40 279

(http://iopscience.iop.org/1751-8121/40/2/007)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **40** (2007) 279–288 doi:10.1088/1751-8113/40/2/007

Davies theory for reservoir-induced entanglement in a bipartite system

K Lendi[1] and A J van Wonderen[2]

[1] Institute of Physical Chemistry, University of Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland
[2] Institute of Theoretical Physics, University of Amsterdam, Valckenierstraat 65, NL-1018 XE Amsterdam, The Netherlands

E-mail: carlen@pci.unizh.ch

Received 4 July 2006, in final form 1 November 2006
Published 12 December 2006
Online at stacks.iop.org/JPhysA/40/279

Abstract

Two mutually noninteracting qubits with identical modest coupling to one and the same reservoir are considered. For a given Hamiltonian and uncorrelated initial state, the mathematically rigorous Davies theory of the weak-coupling and van Hove limit provides a unique Markovian quantum master equation where absolutely none of the usually made additional assumptions and further approximations are introduced. Due to completely positive time evolution also no artificial correlations can arise. Numerical solution of the Markovian master equation shows that the qubits become entangled. In a first short time-interval containing one single maximum of entanglement for reservoir temperature $T = 0$, different choices of uncorrelated initial states give rise to a remarkable emergence of entanglement of different degree. The quantitative evaluation is analysed in terms of a measure derived from Wootters concurrence. Selected results show that there are even states that acquire the possible maximum. Particularly those states will show a periodic type of 'collapse and revival' behaviour with exponentially decaying envelope at longer times. This has never been reported so far for noninteracting qubits as mediated by simultaneous coupling to an uncontrollable reservoir. Moreover, even selected uncorrelated mixed states of modest degree of mixture may show a similar behaviour, although less pronounced. For $T > 0$ states with high degree of entanglement at $T = 0$ in the first time-interval still show a gradually reduced value up to a few tenth of Kelvin but for $T \geqslant 33$ K no effects can be observed. Finally, initially entangled states will slowly lose their oscillatory degree, again with exponential envelope, as the bipartite system approaches its stationary final state.

PACS numbers: 03.65.Ud, 03.65.Yz, 03.67.Mn, 05.30.−d, 5.40.−a, 89.70.+c

1. Introduction

The central importance of entanglement for quantum communication has been shown in numerous treatments [1–4]. Many papers are devoted to questions about how to preserve entanglement for some time, how to increase it, or even how to generate it by various methods [5–8]. In most cases it is naturally assumed that during any dynamical evolution of small quantum systems their unavoidable coupling to some environment causes decoherence effects and a related destruction of a desired entanglement. Several proposals how to avoid decoherence or how to escape into so-called decoherence-free subspaces have also been published [9–15].

It is of particular interest to note that also the possibility of the contrary has been investigated, i.e., the mere creation of entanglement out of an almost or completely disentangled initial state through interaction with the environment. Even though this seems to be somewhat counterintuitive it could be shown that the interaction of a large reservoir in an arbitrary mixed initial state with a single qubit in a pure initial state can dynamically induce a nontrivial correlation [16, 17]. More than that, the time-dependent appearance of entanglement has been evident from exact solutions of simple and also more complicated versions of the Jaynes–Cummings model in quantum optics [18–21]. By coupling a bipartite system (two qubits) to a heat bath of harmonic oscillators the non-Markovian dynamics starting from a separable initial state may also induce some entanglement on shorter or, exceptionally, even longer time scales [22]. Whereas the above-mentioned examples involve an essentially non-Markovian dynamics there are also treatments of rather phenomenological Markovian type [23, 24]. Furthermore, in a more rigorous Markovian setting, entanglement formation has been found in a model of two atoms immersed in a thermostat [25] where the time evolution is given by a completely positive quantum dynamical semigroup with infinitesimal generators simply parametrized in terms of spontaneous emission and photon exchange rates. Finally, in a rather general treatment of two noninteracting two-level systems, coupled to a common reservoir, various conditions on input parameters favouring possible dynamical generation of entanglement out of uncorrelated initial states near $t = 0$ are worked out. The underlying completely positive time evolution is based on a suitable parametrization of generators [26].

The present work exclusively starts from a given Hamiltonian for two noninteracting qubits coupled to a reservoir with well-defined properties in terms of time-dependent correlation functions, as used in common versions of spin-boson models in the ohmic case. In contrast to earlier work, absolutely no additional approximations or further simplifications are introduced such as, e.g., assumptions on vanishing Hamiltonian shifts, or a drastic reduction of the number of relevant relaxation parameters. For this purpose, the only general, mathematically and quantum-mechanically consistent Markovian dynamics in the weak-coupling and van Hove limit is given by the theory of Davies [27]. As will be shown, this leads to a genuine reduced dynamics free of any artificial correlations, as they might be present in phenomenological master equations. The result is a time evolution which, according to all necessary and sufficient requirements [28, 29], is given by a so-called completely positive quantum-dynamical map. This allows us to trace out a quantification of entanglement as a function of time over the entire time scale by using Wootters concurrence and, in particular, offers the possibility of analysing the influence of reservoir temperatures. The analysis of selected cases covers mostly pure, uncorrelated initial states of the qubits at $T = 0$ and $T > 0$, as well as initially entangled states at $T = 0$.

Last, a short summary of the paper is given. In section 2 the Davies theory of quantum dissipation is worked out for the case of two independent, mutually noninteracting qubits that interact with a single reservoir. On the basis of a standard spin-boson model the reservoir pair-correlation function is explicitly evaluated. The latter provides all damping constants and level shifts figuring in the Markovian equation. Section 3 is devoted to the numerical computation of the density matrix that describes the evolution of the bipartite system. The emergence of entanglement is investigated with the help of a measure based on Wootters concurrence. Dependence on the choice of initial states and on reservoir temperatures is analysed. Section 4 contains a discussion of the results.

2. The Davies generator for completely positive time evolution

On the tensor spaces $\mathcal{H} = \mathcal{H}_B \otimes \mathcal{H}_R$, where $\mathcal{H}_B = \mathcal{H}_1 \otimes \mathcal{H}_2$ refers to the two-qubit bipartite system and $\mathcal{H}_R$ to the reservoir, the total Hamiltonian reads

$$\begin{aligned} &H = H_B \otimes \mathbb{1}_R + \mathbb{1}_B \otimes H_R + \lambda H_{BR} \otimes V_R, \\ &H_B = h_Q \otimes \mathbb{1} + \mathbb{1} \otimes h_Q, \qquad h_Q = \tfrac{1}{2}(\Delta\sigma_1 + \varepsilon\sigma_3), \\ &H_{BR} = h_{BR} \otimes \mathbb{1} + \mathbb{1} \otimes h_{BR}, \qquad h_{BR} = \tfrac{1}{2}(w\sigma_1 + u\sigma_3), \end{aligned} \tag{1}$$

H_{BR} provides identical coupling of both mutually noninteracting qubits to the reservoir, and $\{\sigma_1, \sigma_3\}$ are Pauli matrices. The reservoir consists of harmonic oscillators with Hamiltonian $H_R = \sum_n \omega_n a_n^* a_n$ and a coupling term $V_R = \sum_n c_n(a_n^* + a_n)$, to be specified in more detail below in terms of a two-point correlation function. Unitary time-evolution of the total system with density operator $\mathcal{W}_t$ is then given by $\dot{\mathcal{W}}_t = -\mathrm{i}[H, \mathcal{W}_t]$, $\mathcal{W}_t = U_t \mathcal{W}_0 U_{-t}$, $U_t = \mathrm{e}^{-\mathrm{i}Ht}$, starting from a completely uncorrelated initial state $\mathcal{W}_0 = \rho(0) \otimes \mathcal{R}(\beta)$. The reduced density operator for the bipartite system is obtained by the partial trace $\rho(t) = \mathrm{Tr}_{\mathrm{R}}\{\mathcal{W}_{\mathrm{t}}\}$, and the reservoir is supposed to be in the stationary Gibbs state $\mathcal{R}(\beta) = \mathrm{e}^{-\beta H_R}/Z(\beta)$, $\beta = 1/(k_B T)$. According to Davies [27, 30], the Markovian quantum master equation is then given by the following expressions,

$$\dot{\rho}(t) = (D_H + D_R)\{\rho(t)\}, \tag{2}$$

$$D_H\{\rho(t)\} = -\mathrm{i}\left[\left(H_B + \sum_{k,l=1}^{4} \tilde{s}_\beta(\Omega_{kl}) A_{kl}^* A_{kl}\right), \rho(t)\right], \tag{3}$$

$$D_R\{\rho(t)\} = \frac{1}{2}\sum_{k,l=1}^{4} \tilde{c}_\beta(\Omega_{kl})([A_{kl}\rho(t), A_{kl}^*] + [A_{kl}, \rho(t) A_{kl}^*]). \tag{4}$$

A_{kl} is an operator-valued Fourier coefficient in a decomposition of the Heisenberg representation $H_{BR}(t)$ with respect to the difference spectrum of H_B,

$$H_{BR}(t) = U_{-t}^B H_{BR} U_t^B = \sum_{k,l=1}^{4} A_{kl}\, \mathrm{e}^{-\mathrm{i}\Omega_{kl} t}, \qquad U_t^B = \mathrm{e}^{-\mathrm{i}H_B t}, \tag{5}$$

$$H_B = \sum_{m=1}^{4} \lambda_m P_m, \qquad A_{kl} = P_k H_{BR} P_l, \qquad \Omega_{kl} = \lambda_l - \lambda_k. \tag{6}$$

The explicit model eigenvalues are $\lambda_1 = \lambda_2 = 0$, $\lambda_3 = -\lambda_4 = \sqrt{\varepsilon^2 + \Delta^2}$, and $\{P_m\}_1^4$ are the one-dimensional orthogonal projectors. The coefficients in D_R and D_H are given by Fourier and Hilbert transforms of the correlation function $C_\beta(t)$ with respect to the stationary reservoir state $\mathcal{R}(\beta)$,

$$
\begin{aligned}
&C_\beta(t) = \mathrm{Tr}_R(V_R V_R(t)\mathcal{R}(\beta)), \qquad V_R(t) = U^R_{-t} V_R U^R_t,\\
&U^R_t = \mathrm{e}^{-\mathrm{i}H_R t}, \qquad \tilde{c}_\beta(\omega) = \int_{-\infty}^{\infty} C_\beta(t)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t,\\
&\tilde{s}_\beta(\omega) = \frac{\mathcal{P}}{2\pi}\int_{-\infty}^{\infty}\frac{\tilde{c}_\beta(\nu)}{\nu-\omega}\,\mathrm{d}\nu = \mathrm{i}\int_0^{\infty} C_\beta(t)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t - \frac{\mathrm{i}}{2}\tilde{c}_\beta(\omega),
\end{aligned} \tag{7}
$$

where $\mathcal{P}$ denotes the principal part. As frequently used for spin-boson models in the limit of an infinite heat bath in the ohmic case, we choose a smooth, continuous spectral density $J(\nu)$ for the representation of the correlation function [31, 32],

$$
C_\beta(t) = \int_0^{\infty} J(\nu)[\coth(\beta\nu/2)\cos(\nu t) + \mathrm{i}\sin(\nu t)]\,\mathrm{d}\nu, \qquad J(\nu) = \frac{\alpha}{2}\nu\,\mathrm{e}^{-\nu/\omega_c}, \tag{8}
$$

with strength α and cutoff ω_c. The exact result for the time- and temperature-dependent complex-valued correlation function $C_\beta(\tau)$ is then obtained as

$$
C_\beta(\tau) = f_\beta(\tau) + \mathrm{i}g_\beta(\tau), \tag{9}
$$

$$
f_\beta(\tau) = \frac{\alpha\omega_c^2}{2}\frac{1-\tau^2}{(1+\tau^2)^2} + \alpha\omega_c^2\sum_{k=1}^{\infty}\frac{(1+\beta\omega_c k)^2-\tau^2}{[(1+\beta\omega_c k)^2+\tau^2]^2}, \tag{10}
$$

$$
g_\beta(\tau) = \alpha\omega_c^2\frac{\tau}{(1+\tau^2)^2}, \qquad \tau = \omega_c t. \tag{11}
$$

This leads to an exact representation for the Fourier transform of the dissipative contribution, satisfying the indispensable KMS-condition $\tilde{c}_\beta(-\omega) = \mathrm{e}^{-\beta\omega}\tilde{c}_\beta(\omega)$ [27, 29, 33],

$$
\tilde{c}_\beta(\omega) = \frac{\pi\alpha}{2}\,\mathrm{e}^{-\frac{|\omega|}{\omega_c}}\left\{|\omega|\frac{\mathrm{e}^{\beta|\omega|}+1}{\mathrm{e}^{\beta|\omega|}-1} + \omega\right\}, \tag{12}
$$

whereas the Hilbert transform (7) cannot be analytically evaluated and must be obtained by numerical computation.

In order to show complete positivity with help of a representation derived by Gorini, Kossakowski and Sudarshan [28, 29] one transforms the above equations into normal form,

$$
\dot{\rho}(t) = (D_H + D_R)\{\rho(t)\} \to \mathcal{L}_H\{\rho(t)\} + \mathcal{L}_R\{\rho(t)\}. \tag{13}
$$

In particular, the relevant dissipative contribution is represented by

$$
\mathcal{L}_R\{\rho(t)\} = \frac{1}{2}\sum_{i,k=1}^{15} a_{ik}([F_i, \rho(t)F_k] + [F_i\rho(t), F_k]), \qquad \mathcal{A} = \{a_{ik}\}_{i,k=1}^{15} \geqslant 0, \tag{14}
$$

where $\mathcal{A}$ must be positive-semidefinite and the set $\{F_i\}_1^{15}$ is a complete orthonormal matrix set, conveniently given by the infinitesimal generators of the Lie algebra of $SU(4)$. For the transformation $D_R \to \mathcal{L}_R$ we use the expansion

$$
A_{mn} = \sum_{i=1}^{15}\mu_i^{mn}F_i + \mu_0 \mathbb{1}_B, \qquad \mu_i^{mn} = \mathrm{Tr}(A_{mn}F_i), \tag{15}
$$

$$
a_{ik} = \sum_{m,n=1}^{4}\tilde{c}_\beta(\Omega_{mn})\mu_i^{mn}\overline{\mu_k^{mn}}, \tag{16}
$$

where the bar denotes complex conjugation. The above structure of a_{ik} guarantees that $\mathcal{A} = \mathcal{A}^*$ and $\mathcal{A} \geqslant 0$, such that it is explicitly shown that the Davies form generates a completely positive quantum dynamical semigroup.

3. Method of solution and measures of entanglement

The numerical solution is then performed in a suitable coherence-vector representation leading to 15 coupled, real first-order differential equations, as obtained from the decomposition

$$\rho(t) = \frac{1}{4}\mathbb{1}_B + \sum_{i=1}^{15} v_i(t) F_i, \qquad \rho(0) = \rho_1 \otimes \rho_2, \tag{17}$$

$$\dot{\rho}(t) \to \dot{\vec{v}}(t) = (Q + R)\vec{v}(t) + \vec{k}, \tag{18}$$

with an uncorrelated pure initial state for both qubits. All details for the transformations $\mathcal{L}_H \to Q$ and $\mathcal{L}_R \to \{R, \vec{k}\}$ are available elsewhere [29] but, for completeness, necessary formulae are summarized in the appendix.

In order to investigate effects of a variation of initial states we will choose a few relevant examples from the representation

$$\rho_i(\varphi) = \begin{pmatrix} \cos^2\varphi & \sin\varphi\cos\varphi \\ \sin\varphi\cos\varphi & \sin^2\varphi \end{pmatrix}, \qquad i = 1, 2, \tag{19}$$

such that, e.g., for $\varphi = 0$ both qubits are in their upper state and for $\varphi = \pi/2$ they are in the lower state.

In principle, the manifestation of entanglement can be tested by various methods. Perhaps the first striking example shows up in the complete Nakajima–Zwanzig equation for non-Markovian reduced dynamics. For non-factorizable initial states the difference $J = \rho(0) - \rho_1(0) \otimes \rho_2(0)$, where $\rho_1 = \mathrm{Tr}_2(\rho)$, $\rho_2 = \mathrm{Tr}_1(\rho)$, gives rise to a nasty inhomogeneous term which causes additional serious mathematical difficulties and, in fact, excludes any chance for semigroup dynamics [34]. A very interesting more recent proof shows that entanglement is uniquely related to a negative eigenvalue of the density matrix after partial transposition in one of the qubit components in a Kronecker product representation of ρ in terms of Pauli matrices [26, 35, 36]. Here, preference will be given to the Wootters measure [37] for a commonly used quantification of entanglement, denoted by $E_W(t)$. The definition is as follows,

$$\begin{aligned} &E_W(t) = \mathcal{F}\big(\tfrac{1}{2}\big[1 + (1 - \Phi^2)^{\frac{1}{2}}\big]\big), \\ &\mathcal{F}(y) = -y\log_2(y) - (1-y)\log_2(1-y), \\ &\Phi = \max\{0, 2\lambda_{\max}(\hat{\rho}) - \mathrm{Tr}(\hat{\rho})\} : \text{concurrence}, \\ &\hat{\rho} = \big(\rho^{\frac{1}{2}}\tilde{\rho}\rho^{\frac{1}{2}}\big)^{\frac{1}{2}}, \qquad \tilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^T(\sigma_2 \otimes \sigma_2). \end{aligned} \tag{20}$$

Above, ρ^T is the ordinary transpose, $\lambda_{\max}$ the largest eigenvalue of $\hat{\rho}$ and σ_2 again a Pauli matrix. Note that $E_W = 0$ is obtained for separable states, whereas the maximally entangled singlet state of two spin-(1/2) systems, e.g., acquires the value $E_W = 1$, such that for all states $\{0 \leqslant E_W \leqslant 1\}$ is satisfied.

The first considerations regard the dynamics at temperature $T = 0\,(\beta = \infty)$. As shown in figure 1, depending on the choice of inital state a remarkable emergence of entanglement builds up to about $t = 0.6$, followed by a subsequent decrease. Surprisingly, even almost complete entanglement is achieved for $\varphi \approx 0.4$ whereas, by symmetry, the values for $\varphi = 0$ and $\varphi = \pi/2$ are identical.

Of course, the coupling to the reservoir is expected to induce an irreversible process with a stationary final state reached in the infinite time limit. In fact, due to the particular, efficient coupling H_{BR} the quantum dynamical semigroup is uniquely relaxing, that is, any arbitrary initial state ends in one and the same final destination state [29] and is, of course, independent

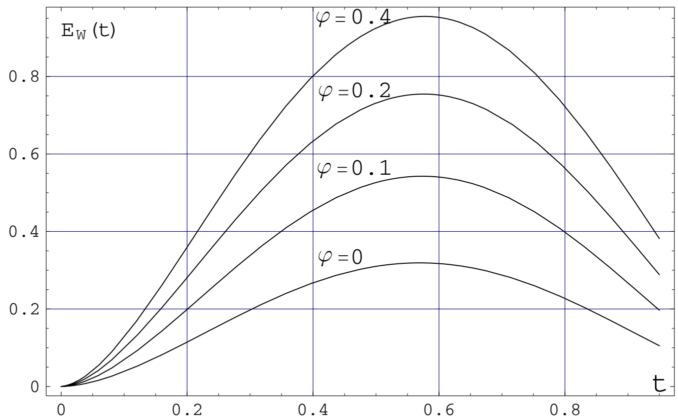

Figure 1. Degree of entanglement $E_W(t)$ according to (20), in a first interval and for different initial states according to (19). The model parameters have been set to $\varepsilon = \Delta = w = 2$, $u = 4$, $\alpha = 0.01$, $\omega_c = 30$, $T = 0$.

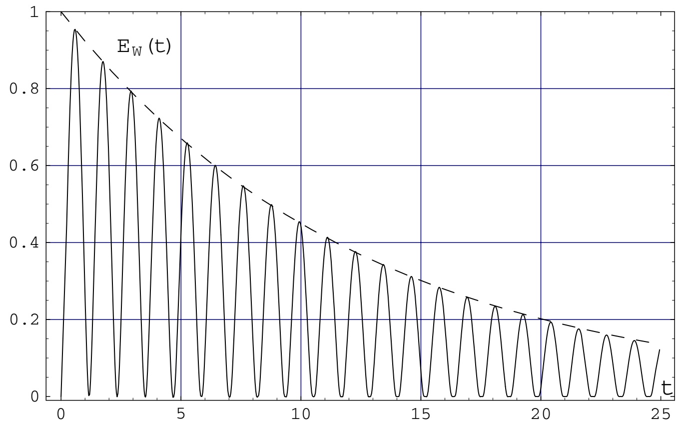

Figure 2. Oscillating longtime behaviour of generated entanglement $E_W(t)$ with a perfect exponential envelope $e^{-t/\tau}$, $\tau = 12.5$, for $\varphi = 0.4$ and $T = 0$.

of any choice of φ. Therefore, one gets $\rho(\infty)$ from $\vec{v}(\infty) = -(Q + R)^{-1}\vec{k}$ in (18). In particular, the final state turns out to be pure and uncorrelated, i.e., $\rho(\infty) = \rho^2(\infty)$, $\rho(\infty) = \rho^{(1)}(\infty) \otimes \rho^{(2)}(\infty)$, where each factor is also a pure state. Thus, time-evolution causes two initially disentangled pure states to become entangled for a finite period whereas in the asymptotic regime they become disentangled and pure again. As can be seen in more detail on the longer time scale in figure 2, a periodic 'collapse and revival' of entanglement occurs with an exponentially decreasing amplitude envelope of a relatively long lifetime. This remains true for $t > 25$ and can numerically be calculated to arbitrary desired precision at any time but, as stated above, the final state is more easily obtained by inverting $(Q + R)$.

A further question concerns the role of purity in the initial states and a related possible disappearance of entanglement upon increasing degree of mixture. In fact, the maximum of the $(\varphi = 0)$-peak in figure 1 decreases almost linearly as a function of x for a choice $\rho_1 = \rho_2 = \mathrm{diag}(x, 1 - x)$ and, therefore, $\rho = \mathrm{diag}(x^2, x(1 - x), x(1 - x), (1 - x)^2)$ for the bipartite initial state. In detail, one finds $E_W = 0$ for $x \geqslant 0.23$, and no entanglement will

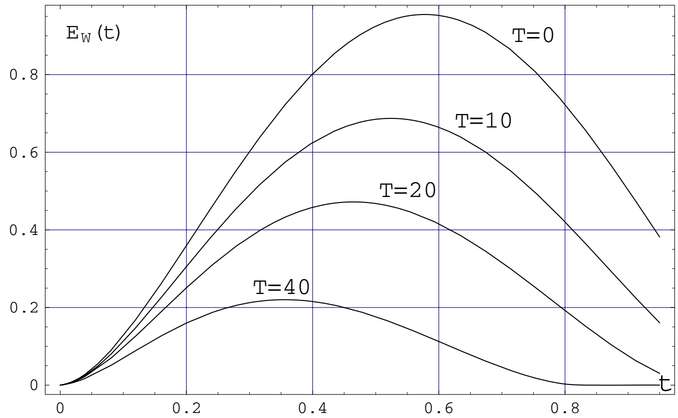

Figure 3. Comparison of four temperatures in the first interval, for model parameters as in figure 1, $\beta = 1/T$ and $\varphi = 0.4$.

show up for larger values of x. But the latter value corresponds to about 77% of the maximum of the single qubit entropy. This shows that there is quite an appreciable range of mixed states leading to dynamic entanglement but, for $x \geqslant 0.1$ the peaks appear only in the first short interval $0 \leqslant t \leqslant 1$.

Now, for $T > 0$, we set $\beta = 1/T$, consistent with the chosen model parameters but, again, the particularly pronounced case $\varphi = 0.4$ will be investigated. One may then expect a gradually decreasing degree of entanglement with rising temperature. This is confirmed by the results shown in figure 3. To have physically realistic quantities in mind, one can appropriately scale the used parameters. For instance, for an electron spin as a single qubit with an energy splitting of 28 GHz in 1 Tesla, one obtains a time unit of 0.1 ns and the same Boltzmann factors as for our dimensionless model parameters if the three temperature values $\{T = 10, 20, 40\}$ in figure 3 are reduced by a factor of 0.475. Thus, at equilibrium with the reservoir, the physical temperatures for a two-level system are {4.75 K, 9.5 K, 19 K} with corresponding ratios {0.75, 0.87, 0.93} between occupation numbers of upper to lower level.

This shows that entanglement occurs even at temperatures which are not extremely low, but essentially restricted to a first short time interval only. In fact, the situation shown in figure 2 is unique for $T = 0$, and for $T > 0$ no reappearance of entanglement at later times is observed, except for $T = 10$ with an appreciably smaller second and even much smaller third peak. And, finally, any entanglement vanishes completely for $T > 70$ ($\approx$ 33 K).

Of course, due to complete positivity the method allows to study any type of initial states, and one may wonder about the fate of states with initial degree of entanglement $0 < E_W(0) \leqslant 1$. First of all, the states used for figure 1 show clearly how entangled initial states propagate. This is trivially due to the properties of semigroup time evolution where states at any time $t_0 > 0$ can be taken as initial states with identical subsequent evolution [29]. Nevertheless, we analyse in addition a few other especially prepared states derived directly from the singlet,

$$\rho_S = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad E_W = 1. \tag{21}$$

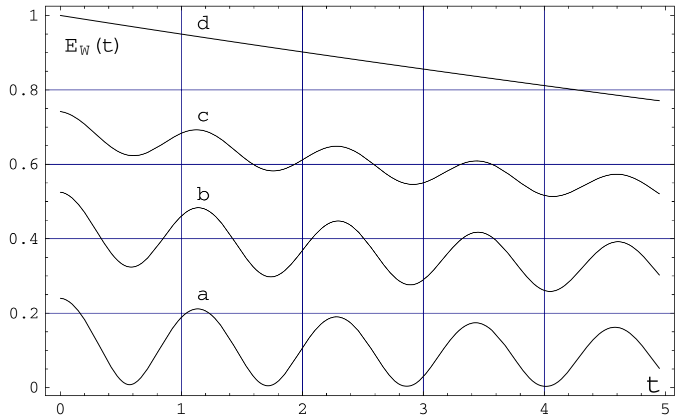

Figure 4. Evolution of states with entanglement at $t = 0$ and $T = 0$. a: $E_W(0) = 0.240$; b: $E_W(0) = 0.525$; c: $E_W(0) = 0.742$; d: $E_W(0) = 1$.

ρ_S is then suitably modified by parametrized orthogonal transformations according to

$$\rho_S \rightarrow \tilde{\rho}_S = D\rho_S D^T, \qquad DD^T = \mathbb{1}_4. \tag{22}$$

A convenient choice for D is offered by the special Wigner matrices for spin-$(3/2)$ [38] with appropriate variation of the polar angle. Figure 4 shows the result of a few cases with different initial degree of entanglement, as generated from (22). The straight line (d) puts in evidence why the extremal singlet state may be called robust since it goes slowly to zero without any oscillations. Not shown is an exponential envelope similar to that in figure 2 which here would have a somewhat longer decay time of about $\tau \approx 17$, but almost identically for all states.

4. Conclusion

In conclusion, we have shown that a bipartite system, starting from uncorrelated pure initial states of two mutually noninteracting qubits, can acquire considerable entanglement, even up to its maximum value possible, during its Markovian time evolution, as due to some coupling to the environment. For a reliable quantification of the degree of entanglement Wootters concurrence has proven to be particularly appropriate. The methods used are able to treat initially uncorrelated as well as correlated states at different temperatures $T \geqslant 0$ and on the entire time-scale $t \geqslant 0$, including the limit $t \rightarrow \infty$.

The generation of entanglement shows a sensitive dependence on temperature. In particular, the initially oscillating amplitude is gradually reduced, essentially to a first peak only, as soon as temperature rises. It is important to stress again that the results follow exclusively on the basis of well-defined Hamiltonians and initial conditions, and no tentative and rather artificial additional assumptions have been introduced such as, e.g.,vanishing Hamiltonians or very special, purely phenomenological relaxation parameters. In fact, the consequent calculation and inspection of the relaxation matrix $\mathcal{A}$ shows that, except for a very few ones, all elements are of essential magnitude, and it would be completely unreasonable to introduce any unnecessary simplifications. And furthermore, the fundamental property of complete positivity excludes any artificial entanglement, as it may occur when using phenomenological master equations [39], nor is the analysis restricted to any special class of initial states. Finally, for the same model as used in the present work, preliminary calculations on non-Markovian dynamics were carried out with help of a very accurate code [40, 41] showing a very satisfactory agreement with the Markovian results. Therefore, it is reasonable to expect that our findings possess quite general validity, as long as the coupling between qubits and reservoir remains of moderate magnitude.

Acknowledgments

Support by the Swiss National Science Foundation SNF, Project No 200020-101608, is gratefully acknowledged.

Appendix

The infinitesimal generators of $SU(4)$ satisfy the following relations,

$$F_j = F_j^*, \qquad \mathrm{Tr}(F_j) = 0, \qquad \mathrm{Tr}(F_j F_k) = \delta_{jk}, \tag{A.1}$$

$$[F_j, F_k] = \mathrm{i}\sum_{l=1}^{15} f_{jkl} F_l, \qquad \{F_j, F_k\} = \frac{1}{2}\mathbb{1}_B \delta_{jk} + \sum_{l=1}^{15} d_{jkl} F_l, \tag{A.2}$$

$$F_j F_k = \frac{1}{4}\mathbb{1}_B \delta_{jk} + \frac{1}{2}\sum_{l=1}^{15}(d_{jkl} + \mathrm{i} f_{jkl}) F_l. \tag{A.3}$$

The coefficients $\{f_{jkl}\}$ are the completely antisymmetric (with respect to interchanging any pair of indices) and $\{d_{jkl}\}$ the completely symmetric structure constants of the Lie algebra of $SU(4)$. Their values are tabulated in a common representation in [29]. Using (A.2) and (A.3) any multilinear expression in the generators can be contracted to a linear form.

Details for the transformations $\mathcal{L}_H \to Q$ and $\mathcal{L}_R \to \{R, \vec{k}\}$ are available elsewhere [29], but for completeness a summary will be given. The matrix elements $Q = \{q_{ik}\}$ of the Hamiltonian contribution read

$$q_{ik} = \sum_{l=1}^{15} h_l f_{lki}, \qquad h_l = \mathrm{Tr}\left(H_B^{\mathrm{tot}} F_l\right), \tag{A.4}$$

$$H_B^{\mathrm{tot}} = H_B + \sum_{k,l=1}^{4} \tilde{s}_\beta(\Omega_{kl}) A_{kl}^* A_{kl} = \sum_{k=1}^{15} h_k F_k + \frac{1}{4}\mathrm{Tr}\left(H_B^{\mathrm{tot}}\right)\mathbb{1}_B. \tag{A.5}$$

Due to the commutator in (3) only the traceless components of H_B^{tot} are needed.

The matrix elements of the relaxing part $R = \{r_{ik}\}$ and the components of the vector $\vec{k}$ are given by

$$r_{ik} = -\frac{1}{4}\sum_{\substack{l,m,n=1 \\ (m\leqslant n)}}^{15} (2 - \delta_{mn})\,\mathrm{Re}(a_{mn})\{f_{ilm} f_{kln} + f_{iln} f_{klm}\} \tag{A.6}$$

$$+\frac{1}{2}\sum_{\substack{l,m,n=1 \\ (m<n)}} \mathrm{Im}(a_{mn})\{f_{ilm} d_{kln} - f_{iln} d_{klm}\}, \tag{A.7}$$

$$k_i = -\frac{1}{2}\sum_{\substack{k,l=1 \\ (k<l)}} \mathrm{Im}(a_{kl}) f_{ikl}. \tag{A.8}$$

References

[1] Bouwmeester D, Ekert A and Zeilinger A 2000 *The Physics of Quantum Information* (Berlin: Springer)
[2] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[3] Alber G, Beth T, Horodecki M, Horodecki P, Horodecki R, Rötteler M, Weinfurter H, Werner R and Zeilinger A 2001 *Quantum Information* (Berlin: Springer)
[4] Vedral V, Plenio M B, Rippin M A and Knight P L 1997 *Phys. Rev. Lett.* **78** 2275
[5] Cirac J I, Dr W, Kraus B and Lewenstein M 2001 *Phys. Rev. Lett.* **86** 544
[6] Dür W, Vidal G, Cirac J I, Linden N and Popescu S 2001 *Phys. Rev. Lett.* **87** 137901
[7] Zyckowski K, Horodecki P, Horodecki M and Horodecki R 2001 *Phys. Rev.* A **65** 012101
[8] Stelmachovic P and Buzek V 2001 *Phys. Rev.* A **64** 062106
Stelmachovic P and Buzek V 2003 *Phys. Rev.* A **67** 029902
[9] Zurek W H 1982 *Phys. Rev.* D **26** 1862
[10] Beige A, Braun D, Tregenna B and Knight P L 2000 *Phys. Rev. Lett.* **85** 1762
[11] Retamal J C and Zagury N 2001 *Phys. Rev.* A **63** 032106
[12] Lidar D A, Chuang I L and Whaley K B 1998 *Phys. Rev. Lett.* **81** 2594
[13] Lidar D A and Whaley K B 2003 Decoherence-Free Subspaces and Subsystems *Irreversible Quantum Dynamics* (*Lecture Notes in Physics* vol 622) ed F Benatti and R Floreanini (Berlin: Springer)
[14] Alicki R 2003 Controlled quantum open systems *Irreversible Quantum Dynamics* (*Lecture Notes in Physics* vol 622) ed F Benatti and R Floreanini (Berlin: Springer)
[15] Wu L A, Zanardi P and Lidar D A 2005 *Phys. Rev. Lett.* **95** 130501
[16] Bose S, Fuentes-Guridi I, Knight P L and Vedral V 2001 *Phys. Rev. Lett.* **87** 050401
[17] Kim M S, Lee Jinhyoung, Ahn D and Knight P L 2002 *Phys. Rev.* A **65** 040101
[18] Phoenix S J D and Knight P L 1991 *Phys. Rev.* A **44** 6023
[19] Farhadmotamed F, van Wonderen A J and Lendi K 1998 *J. Phys. A: Math. Gen.* **31** 3395
[20] van Wonderen A J and Lendi K 2002 *J. Math. Phys.* **43** 4692
[21] van Wonderen A J and Lendi K 2002 *J. Phys. A: Math. Gen.* **35** 9889
[22] Braun D 2002 *Phys. Rev. Lett.* **89** 277901
[23] Plenio M B, Huelga S F, Beige A and Knight P L 1999 *Phys. Rev.* A **59** 2468
[24] Plenio M B and Huelga S F 2002 *Phys. Rev. Lett.* **88** 197901
[25] Jakobczyk L 2002 *J. Phys. A: Math. Gen.* **35** 6383
[26] Benatti F, Floreanini R and Piani M 2003 *Phys. Rev. Lett.* **91** 070402
[27] Davies E B 1974 *Commun. Math. Phys.* **39** 91
Davies E B 1976 *Math. Ann.* **219** 147
[28] Gorini V, Kossakowski A and Sudarshan E C G 1976 *J. Math. Phys.* **17** 821
[29] Alicki R and Lendi K 1987 *Quantum Dynamical Semigroups and Applications* (*Lecture Notes in Physics* vol 286) (Berlin: Springer)
[30] Gorini V, Frigerio A, Verri M, Kossakowski A and Sudarshan E C G 1978 *Rep. Math. Phys.* **13** 149
[31] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford: Oxford University Press)
[32] DiVincenzo D P and Loss D 2005 *Phys. Rev.* B **71** 035318
[33] Thirring W 1983 *Quantum Mechanics of Large Systems* vol 4 (Wien: Springer)
[34] Lendi K 2003 Selected aspects of Markovian and non-Markovian quantum master equations *Irreversible Quantum Dynamics* (*Lecture Notes in Physics* vol 622) ed F Benatti and R Floreanini (Berlin: Springer)
[35] Werner R F 1989 *Phys. Rev.* A **40** 4277
[36] Peres A 1996 *Phys. Rev. Lett.* **77** 1413
[37] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245
[38] Varsalovich D A, Moskalev A N and Khersonskii V K 1988 *Quantum Theory of Angular Momentum* (Singapore: World Scientific)
[39] Alicki R, Lidar D A and Zanardi P 2006 *Phys. Rev.* A **73** 052311
[40] Aissani A and Lendi K 2003 *J. Stat. Phys.* **111** 1353
[41] Kocian P 2006 Impact of von Neumann conditions on non-Markovian dynamics of open quantum systems *PhD Thesis* University of Zürich